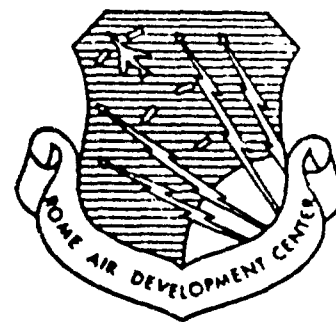


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TRANSIENT AND STEADY-STATE ANTENNA PATTERN  
CHARACTERISTICS FOR ARBITRARY TIME SIGNALS

Fung-I Tseng  
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**TRANSIENT AND STEADY-STATE ANTENNA PATTERN  
CHARACTERISTICS FOR ARBITRARY TIME SIGNALS**

**Fung-I Tseng  
David K. Cheng**

## FOREWORD

This tenth interim report was prepared under Contract No. AF30(602)-2646, Project No. 4506, and Task No. 450604 by Syracuse University Research Institute, Electrical Engineering Department, Syracuse, New York. The contractor's report number is EE957-6503T10.

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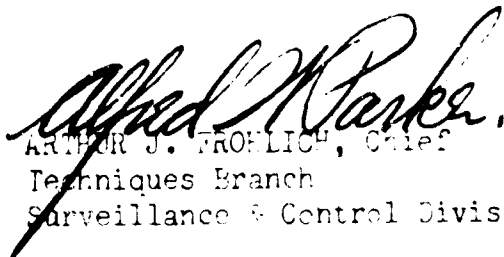
This technical report has been reviewed and is approved.

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## ABSTRACT

This report presents a simplified method for the analytical determination of the transient and steady-state pattern characteristics of an antenna for arbitrary time signals and arbitrary aperture distributions. For uniform aperture illumination, the pattern response of an antenna for any time signal, no matter how complicated, can be found through simple shifts of the signal, and both the transient and the steady-state behavior can be determined without performing any integration. Examples are given for responses to suddenly applied excitations, to rectangular pulses, to excitations for electronic scanning, and to time signals in a pulse-compression system. For pulse signals, a steady state is not reached in any given direction,  $\theta$ , if the duration of the pulse  $T$  is less than  $(a/c) \sin \theta$ , where  $a$  is maximum aperture dimension and  $c$  is velocity of light. Graphical representations for easy interpretation of the different states of pattern characteristics are included.

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TRANSIENT AND STEADY-STATE ANTENNA PATTERN CHARACTERISTICS  
FOR ARBITRARY TIME SIGNALS

by

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I. INTRODUCTION

In the consideration of antenna systems it is generally assumed that pattern characteristics are established instantaneously upon excitation. However, in many modern applications the use of antennas of huge dimensions and time pulses of very short duration creates a situation where the transient behavior is of practical importance. Because electromagnetic waves travel with a finite velocity, the effects at a point in the far zone due to excitations in different parts of a large antenna are not simultaneous. Not until a steady state is reached can we meaningfully discuss pattern characteristics such as beamwidth, sidelobe level, etc.

The transient behavior of aperture antennas has been studied by Polk.<sup>1</sup> It was pointed out that the transient conditions in the radiation pattern of an antenna depend upon the antenna size, the carrier frequency and duration of the time signal, and the amplitude and phase distributions over the antenna aperture. On account of the necessity of evaluating infinite integrals by the theory of residues, Polk's procedure is relatively involved even for the simplest types of time signal and aperture distribution, and, unless great care is exercised, it may lead to erroneous results.<sup>2</sup> For more complicated time signals and aperture distributions, the integral expressions tend to be such that their evaluation will become extremely difficult, if not impossible.

This report presents a simplified approach for the analytical determination of the transient and steady-state pattern characteristics of an antenna for arbitrary time signals and arbitrary aperture distributions. It is not necessary to take the Fourier transform of the time signal or to evaluate complicated inverse transforms. It will be shown that for a uniform aperture distribution the pattern response of an antenna for any time signal can be found through simple shifts of the signal; both the transient and the steady-state behavior can be determined without performing any integration. Examples are given which will show the comparative ease



with which some of the results of Polk and Mayo<sup>1,2</sup> can be obtained.

Of special interest are antenna pattern responses to excitations for electronic scanning and to time signals used in a pulse-compression system. These situations can be handled without difficulty with the present method. In electronic scanning one must be sure that the rate at which the main beam of the radiation pattern reaches its steady-state be faster than that at which the beam scans. In a pulse-compression system, the signal is both time-limited and frequency-modulated and it is of practical interest to determine antenna pattern responses to such signals. It will be shown that there are in general five different states of pattern responses to pulse signals and that the boundaries of these states can be visualized easily with the aid of a graphical representation.

## II. THE BASIC FORMULATION

In this section we shall derive the basic equation which holds for a current function

$$i(x,t) = A(x)f(t), \quad (1)$$

where  $A(x)$  describes an arbitrary space distribution and  $f(t)$  is an arbitrary function of time. Only one-dimensional space variation is considered; extension of the results to the two-dimensional case is trivial when the space distribution is separable in the two dimensions.

Let  $I(x,\omega)$  be the Fourier transform of  $i(x,t)$  in (1).

Then

$$I(x,\omega) = A(x)F(\omega) = \int_{-\infty}^{\infty} i(x,t) e^{-j\omega t} dt \quad (2)$$

and

$$i(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} I(x, \omega) e^{j\omega t} d\omega. \quad (3)$$

The far-zone electric field due to a single-frequency source  $A(x)\exp(j\omega t)$  is given by

$$E_{\omega}(\theta, [t]) = \frac{j\omega\mu_0}{4\pi R} e^{j\omega[t]} \int_{-\infty}^{\infty} A(x) e^{j(\omega x/c) \sin \theta} dx, \quad (4)^*$$

where  $R$  is the distance to the center ( $x = 0$ ) of the antenna,  $c$  is the velocity of light,  $\theta$  is the angle measured from the broadside direction, and  $[t]$  is the retarded time:

$$[t] = t - \frac{R}{c}. \quad (5)$$

For the arbitrary source distribution  $i(x, t)$  the radiation field is, from (2) and (4),

$$\begin{aligned} E(\theta, [t]) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E_{\omega}(\theta, [t]) F(\omega) d\omega \\ &= \frac{\mu_0}{4\pi R} \int_{-\infty}^{\infty} dx \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega I(x, \omega) e^{j\omega([t] + \frac{x}{c} \sin \theta)} d\omega \right\}. \end{aligned} \quad (6)$$

Noting from (3) that

$$\begin{aligned} \left. \frac{\partial i(x, t)}{\partial t} \right|_{t = [t] + \frac{x}{c} \sin \theta} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega I(x, \omega) e^{j\omega([t] + \frac{x}{c} \sin \theta)} d\omega \\ &= \frac{\partial}{\partial [t]} i(x, [t] + \frac{x}{c} \sin \theta), \end{aligned} \quad (7)$$

---

\* This expression is different from that used by Polk who considered an aperture field excitation. Eq. (4) holds for a current source (see reference [3]) and for large source dimensions such that the angle of interest is small ( $\cos \theta \approx 1$ ).

we write (6) as

$$E(\theta, [t]) = \frac{\mu_0}{4\pi R} \int_{-\infty}^{\infty} \frac{\partial}{\partial [t]} f\left([t] + \frac{x}{c} \sin \theta\right) dx. \quad (8)$$

Putting (1) in (8), we have

$$E(\theta, [t]) = \frac{\mu_0}{4\pi R} \int_{-\infty}^{\infty} A(x) \frac{\partial}{\partial [t]} f\left([t] + \frac{x}{c} \sin \theta\right) dx. \quad (9)$$

Equation (9), which is not in the form of a Fourier integral, is a basic relation that will facilitate subsequent considerations.

### III. GENERAL BEHAVIOR OF FINITE APERTURES

It is obvious that (9) can be written alternatively as

$$E(\theta, [t]) = \frac{\mu_0 c}{4\pi R \sin \theta} \int_{-\infty}^{\infty} A(x) \frac{\partial}{\partial x} f\left([t] + \frac{x}{c} \sin \theta\right) dx. \quad (10)$$

Integrating (10) by parts and noting that, for finite apertures,  $A(\infty) = A(-\infty) = 0$ , we obtain

$$E(\theta, [t]) = -\frac{\mu_0 c}{4\pi R \sin \theta} \int_{-\infty}^{\infty} A'(x) f\left([t] + \frac{x}{c} \sin \theta\right) dx, \quad (11)$$

where  $A'(x) = dA(x)/dx$ , and  $f(t)$  is bounded for all  $t$ .

If we have a uniform source distribution over the finite aperture,  $-a/2 \leq x \leq a/2$ ,

$$\begin{aligned} A(x) &= \text{Rect} \left( \frac{x}{a} \right) \\ &= \begin{cases} 1, & \text{for } -a/2 < x < a/2 \\ 0, & \text{elsewhere,} \end{cases} \end{aligned} \quad (12)$$

then

$$A'(x) = \delta\left(x + \frac{a}{2}\right) - \delta\left(x - \frac{a}{2}\right). \quad (13)$$

Substitution of (13) in (11) yields the integrated result by inspection:

$$E(\theta, [t]) = - \frac{\mu_0 c}{4\pi R \sin \theta} \left\{ f([t] - \frac{a}{2c} \sin \theta) - f([t] + \frac{a}{2c} \sin \theta) \right\}. \quad (14)$$

By writing

$$u = \frac{a}{2c} \sin \theta, \quad (15)$$

we obtain the following simple expression for the far-zone electric field of an uniform line source in which the current varies as an arbitrary function of time  $f(t)$ :

$$E(u, [t]) = \frac{\mu_0 a}{8\pi R u} \left\{ f([t] + u) - f([t] - u) \right\}. \quad (16)$$

Reciprocity relationship tells us that (16) also represents the response of a receiving antenna to an arbitrary time signal  $f(t)$ . Note that (16) is expressed in terms of the given time function, requires no integrations, and gives the response in a simple manner no matter how complicated  $f(t)$  is. Both transient and steady-state behaviors can be determined from (16).

If

$$f(t) = e^{j\omega_0 t}, \quad (17)$$

it is readily verified that (16) yields the well-known form of steady-state, single-frequency response:

$$E_0(\theta, [t]) = \frac{j\omega_0 \mu_0 a}{4\pi R} e^{j\omega_0 [t]} \left\{ \frac{\sin \omega_0 u}{\omega_0 u} \right\}. \quad (18)$$

For an antenna array of  $N$  discrete, equally spaced (spacing  $d$ ) elements, we may write

$$i(x, t) = f(t) \sum_{m=0}^{N-1} A_s(x) \delta(x - md), \quad (19)$$

where  $A_s(md)$  represents the strength of the discrete current source at  $x = md$ . Substitution of (19) in (8) gives

$$E_N(\theta, [t]) = \frac{\mu_0}{4\pi R} \sum_{m=0}^{N-1} A_s(md) \frac{\partial}{\partial [t]} f([t] + \frac{md}{c} \sin \theta). \quad (20)$$

By setting  $f(t) = \exp(j\omega_0 t)$ , (20) reduces to

$$E_N(\theta, [t]) = \frac{j\omega_0 \mu_0}{4\pi R} e^{j\omega_0 [t]} F_N(\theta), \quad (21)$$

where

$$F_N(\theta) = \sum_{m=0}^{N-1} A_s(md) e^{jm(\omega_0 d/c) \sin \theta} \quad (22)$$

is the well-known pattern function of an N-element discrete array for single-frequency operation.

#### IV. RESPONSES TO SUDDEN EXCITATIONS

For sudden excitations uniformly applied to an antenna of a finite aperture we set

$$i(x, t) = \text{Rect} \left( \frac{x}{a} \right) f_1(t) U(t), \quad (23)$$

where  $U(t)$  is a unit step function applied at  $t = 0$ . Substitution of  $f_1(t)U(t)$  for  $f(t)$  in (16) yields the desired response directly. As an example, let us assume  $f_1(t) = \sin \omega_0 t$ .

$$f(t) = (\sin \omega_0 t) U(t). \quad (24)$$

Substitution of (24) in (16) gives

$$E(u, [t]) = \frac{\mu_0 a}{8\pi R u} \left\{ \sin \omega_0 ([t] + u) \cdot U([t] + u) - \sin \omega_0 ([t] - u) \cdot U([t] - u) \right\} \quad (25)$$

Interpretation of (25) is facilitated by Fig. 1. In Fig. 1(a)

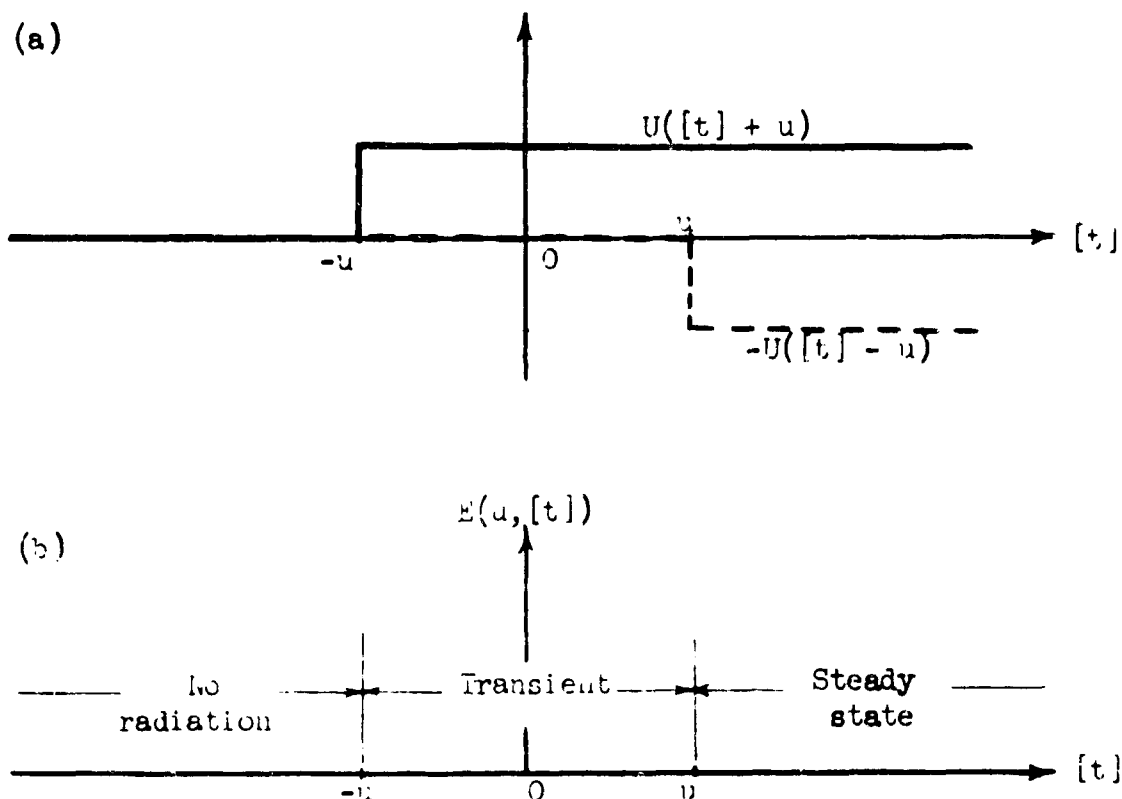


Fig. 1 - Pertaining to response for uniformly excited aperture to suddenly applied sinusoidal time signal.  
 (a) Two step-functions in Eq. (25).  
 (b) Transient and steady-state regions.

are shown the step functions signifying the starting times of the two terms in (25). These two terms combine to give the three distinct regions shown in Fig. 1(b). Writing the expressions for  $E(u, [t])$  in these regions separately, we have

$$E = 0, [t] \leq -u. \quad (26a)$$

$$E = \frac{E_0 u}{\pi R} \cdot \frac{\sin \omega_0([t] + u)}{u}, -u \leq [t] \leq u. \quad (26b)$$

$$E = \frac{E_0 u}{\pi R} \cdot \frac{\sin \omega_0 u}{u} \cos \omega_0 [t], [t] \geq u. \quad (26c)$$

Eqs. (26) check with those given by Mayo.<sup>2</sup> We have obtained them here with a much simpler procedure.

The response to a suddenly applied sinusoidal signal for an antenna with tapered space distribution can also be easily determined and interpreted. For example, for the case of cosine illumination

$$A(x) = \cos\left(\frac{\pi}{a}x\right) \text{Rect}\left(\frac{x}{a}\right) \quad (27a)$$

$$f(t) = (\sin \omega_0 t)U(t), \quad (27b)$$

substitution of (27a) and (27b) in (11) gives the transient and steady-state responses, which check with the corrected versions of Mayo.<sup>2</sup>

#### V. RESPONSES TO RECTANGULAR PULSES

When a rectangular pulse of duration  $T$  superimposed upon a carrier with angular frequency  $\omega_0$  is applied on an antenna of finite dimension  $|x| \leq a/2$ , the transient pattern behavior becomes rather complex since the different interrelationships among the four quantities  $[t]$ ,  $u$ ,  $a$ , and  $T$ , lead to five different combinations. We choose a cosine space distribution to illustrate our method:

$$A(x) = \cos\left(\frac{\pi}{a}x\right) \cdot \text{Rect}\left(\frac{x}{a}\right) \quad (28a)$$

$$f(t) = \sin \omega_0 t \cdot \text{Rect}\left(\frac{t}{T}\right). \quad (28b)$$

Substituting (28a) and (28b) in (11) and noting (15), we have

$$E(u, [t]) = \frac{\mu_0}{8Ru} \int_{x_1}^{x_2} \text{Rect}\left(\frac{x}{a}\right) \sin \omega_0 \left([t] + \frac{2u}{a}x\right) \sin \frac{\pi}{a}x \, dx, \quad (29)$$

$$\text{where} \quad x_1 = -\left([t] + \frac{T}{2}\right) \frac{a}{2u}, \quad (30)$$

$$\text{and} \quad x_2 = -\left([t] - \frac{T}{2}\right) \frac{a}{2u}. \quad (31)$$

Eq. (29) involves only a simple integration over the range  $x_1 \leq x \leq x_2$ .

Because of the function  $\text{Rect}(x/a)$ , the integral will have contribution only when the range of integration overlaps with the range of the finite aperture  $-a/2 \leq x \leq a/2$ . Proper evaluation of (29) depends upon whether  $a$  is greater or less than  $T/2$ . In each case, five different situations may be recognized. ( $u$  is taken to be positive here).

Case I:  $u \leq T/2$   $(x_2 - x_1 = T a/2u \geq a)$ .

(I-a)  $x_1 \geq a/2$ , or  $[t] \leq -(T/2 + u)$ .

$$E_a = 0. \quad (32a)$$

(I-b)  $-a/2 \leq x_1 \leq a/2 \leq x_2$ , or  $-(T/2 + u) \leq [t] \leq -(T/2 - u)$ .

$$E_b = \frac{\mu_o a \omega_o}{4R} \frac{\cos \omega_o([t] + u)}{\pi^2 - 4\omega_o^2 u^2} - \frac{\mu_o a}{16Ru} \left\{ \frac{\sin \left[ \frac{\omega_o T}{2} - \frac{\pi}{2u} \left( [t] + \frac{T}{2} \right) \right]}{\pi - 2\omega_o u} + \frac{\sin \left[ \frac{\omega_o T}{2} + \frac{\pi}{2u} \left( [t] + \frac{T}{2} \right) \right]}{\pi + 2\omega_o u} \right\} \quad (32b)$$

(I-c)  $x_1 \leq -a/2 \leq a/2 \leq x_2$ , or  $-(T/2 - u) \leq [t] \leq T/2 - u$ .

$$E_c = \frac{\mu_o a \omega_o}{2R} \cdot \frac{\cos \omega_o u}{\pi^2 - 4\omega_o^2 u^2} \cos \omega_o [t]. \quad (32c)$$

Note that this is the steady-state region.

(I-d)  $x_1 \leq -a/2 \leq x_2 \leq a/2$ , or  $T/2 - u \leq [t] \leq T/2 + u$ .



$$E_d = \frac{\mu_0 a \omega_0}{4R} \frac{\cos \omega_0([t] - u)}{\pi^2 - 4\omega_0^2 u^2} - \frac{\mu_0 a}{16Ru} \left\{ \frac{\sin \left[ \frac{\omega_0 T}{2} + \frac{\pi}{2u} \left( [t] - \frac{T}{2} \right) \right]}{\pi - 2\omega_0 u} + \frac{\sin \left[ \frac{\omega_0 T}{2} - \frac{\pi}{2u} \left( [t] - \frac{T}{2} \right) \right]}{\pi + 2\omega_0 u} \right\}. \quad (32d)$$

$$(I-e) \quad x_2 \leq -a/2, \quad \text{or} \quad [t] \geq T/2 + u.$$

$$E_e = 0. \quad (32e)$$

Case II:  $u \geq T/2$  ( $x_2 - x_1 \geq a$ ).

The situations here are the same as those for Case I above except for (I-c), since the situation in (I-c) no longer exists. The new conditions are:

$$(II-c) \quad -a/2 \leq x_1 < x_2 \leq a/2, \quad \text{or} \quad -(u - T/2) \leq [t] \leq u - T/2.$$

$$E_c = -\frac{\mu_0 a}{8Ru} \left\{ \frac{\sin \frac{T}{4u} (\pi - 2\omega_0 u)}{\pi - 2\omega_0 u} - \frac{\sin \frac{T}{4u} (\pi + 2\omega_0 u)}{\pi + 2\omega_0 u} \right\} \cos\left(\frac{\pi}{2} \frac{[t]}{u}\right). \quad (33)$$

As contrast to the case in (I-c), (33) does not represent a steady-state situation, which is never reached for  $u > T/2$ . Within the region (II-c), radiation is due to an aperture of a reduced width  $(x_2 - x_1) = Ta/2u$  and lasts for a duration equal to  $(2u - T)$ , during which time the effective part of the aperture shifts from one end of the physical aperture to the other. A graphical interpretation for the transient and steady-state regions of Case I is given in Fig. 2.

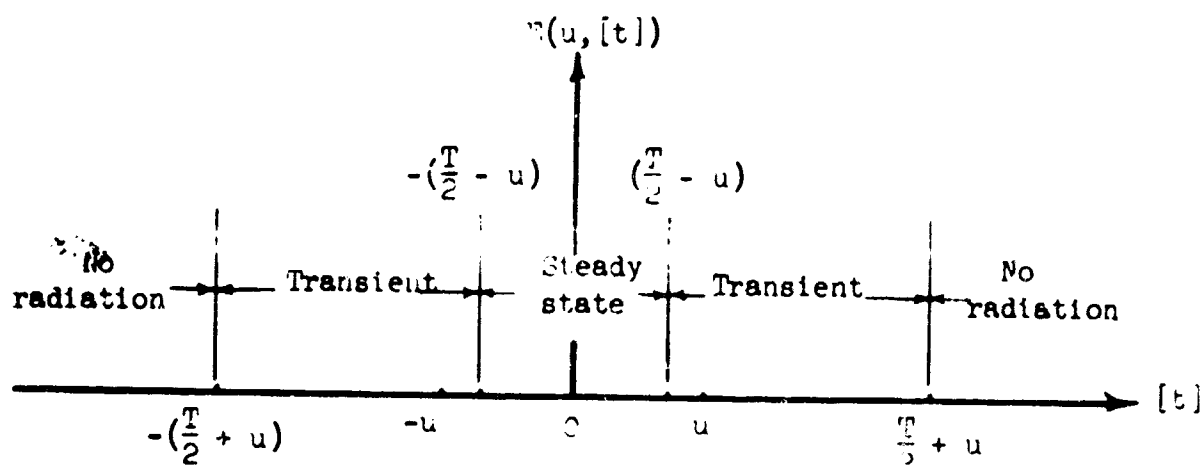


Fig. 2 - Regions of response for aperture with cosine illumination to rectangular pulses of sinusoidal carrier,  $T \geq 2u$ .

## VI. RESPONSES TO EXCITATIONS FOR ELECTRONIC SCANNING

The consideration of the transient behavior of an antenna for electronic scanning is important because of the necessity for ascertaining that the main beam of the radiation pattern reaches its steady state at a rate faster than the rate of scan. For scanning, the excitation function  $i(x, t)$  can not be written in a separable form with respect to  $x$  and  $t$  as in (1). However, the condition of separability was not really required for the derivation of (8); it was a matter of simple superposition of antenna responses to components of the form  $I(x, \omega) \exp(j\omega t)$ . Formal justification of the above statement lies in the essence of solution of inhomogeneous differential equations by Fourier analysis.<sup>4</sup>

Let us consider the case of a discrete, N-element, equispace, phased array as shown in Fig. 3. The elements along the array carry equal currents and are fed through feeders with progressive phase shifts

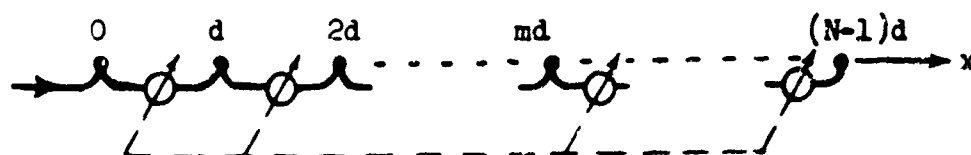


Fig. 3 - An N-element equispace scanning array.

which vary in accordance with a sinusoidal time modulation. We write

$$i(x, t) = I_s \sum_{m=0}^{N-1} \delta(x - md) \cos \omega_0(t - px), \quad (34)$$

where  $I_s$  is the strength of the discrete elements and

$$p = (\beta/\omega_0) \sin \Omega t. \quad (35)$$

$\beta$  in (35) is a system constant which determines the maximum scan angle and

$\Omega$  is the angular frequency of modulation (scanning). Substituting (34)

in (8), we obtain

$$E(\theta, [t]) = \frac{\mu_0 \cos \theta}{4\pi R} \frac{\partial}{\partial [t]} \sum_{m=0}^{N-1} \cos \left\{ \omega_0([t] + \frac{md}{c} \sin \theta) - \Omega m d \sin \Omega ([t] + \frac{md}{c} \sin \theta) \right\}. \quad (36)$$

We have restored the factor  $\cos \theta$  here because we can no longer assume a small angle of interest in case of scanning antennas (see footnote in connection with (4)).

The sine function in (36) can be expanded, and when

$$\left| \frac{\alpha(N-1)d}{c} \sin \theta \right| \ll 1, \quad (37)$$

which is usually satisfied because of the small  $\alpha$ , (36) can be written approximately as

$$E(\theta, [t]) \approx \frac{\mu_0 \cos \theta}{4\pi R} \frac{\partial}{\partial [t]} \sum_{m=0}^{N-1} \cos \left\{ \omega_0 [t] + m\psi \right\} \\ = \frac{\mu_0 \cos \theta}{4\pi R} \frac{\partial}{\partial [t]} \left\{ \frac{\sin (N\psi/2)}{\sin (\psi/2)} \cos \left( \omega_0 [t] + \frac{N-1}{2}\psi \right) \right\}, \quad (38)$$

where

$$\psi = \omega_0 d \left( \frac{1}{c} \sin \theta - \frac{\beta}{\omega_0} \sin \alpha [t] \right). \quad (39)$$

If, in addition,

$$\beta \alpha (N-1)d \ll 2\omega_0, \quad (40)$$

(38) reduces to

$$E(v, [t]) = \frac{\mu_0 \cos \theta}{4\pi R} \left\{ \frac{\sin N\omega_0 \left( v - \frac{\beta d}{2\omega_0} \sin \alpha [t] \right)}{\sin \omega_0 \left( v - \frac{\beta d}{2\omega_0} \sin \alpha [t] \right)} \right\} \\ \cdot \sin \omega_0 \left\{ [t] + (N-1) \left( v - \frac{\beta d}{2\omega_0} \sin \alpha [t] \right) \right\}, \quad (41)$$

$$\text{where} \quad v = (d/2c) \sin \theta. \quad (42)$$

It is clear that  $E(v, [t])$  in (41) represents a pattern with a  $(\sin N\phi)/\sin \phi$  envelope. The main beam of the pattern points in the direction where  $\phi = 0$ , i.e., where

$$v = \frac{\beta d}{2\omega_0} \sin \alpha [t]. \quad (43)$$

The rate of scan can be formed by combining (42) and (43) and taking

derivatives with respect to  $t$ :

$$\frac{d\theta}{dt} = \frac{\beta c a}{\omega_0 \cos \theta} \cos g[t], \quad (44)$$

which, in view of (40), becomes

$$\left| \frac{d\theta}{dt} \right|_{\max.} \ll \frac{2c}{(N-1)d \cos \theta}. \quad (45)$$

Since the right-hand side of this inequality assumes the smallest value when  $\cos \theta$  is unity, we can write (45) as

$$\left| \frac{d\theta}{dt} \right|_{\max.} \ll \frac{2}{\Delta t}, \quad (46)$$

where  $\Delta t$  is the time required for the wave to travel the length of the array. Similarly, (37) can be re-written as

$$g_{\max.} \ll \frac{1}{\Delta t}. \quad (47)$$

Pattern distortion will result if either (46) or (47) is not satisfied.

## VII. RESPONSES TO TIME SIGNALS IN A PULSE-COMPRESSION SYSTEM

Time signals in a pulse-compression system are pulses of long duration with a frequency-modulated carrier.<sup>6</sup> It is of interest to study the transient and steady-state behavior of an antenna in response to such signals. Let us consider

$$i(x, t) = \text{Rect} \left( \frac{x}{a} \right) \text{Rect} \left( \frac{t}{T} \right) \cos (\omega_0 t + \epsilon t^2), \quad (48)$$

which represents a pulse-compression ratio  $= \epsilon T^2/x$ . By (16), we find the response directly.

$$\begin{aligned} E(u, [t]) = \frac{u_0 a}{8\pi R u} & \left\{ \text{Rect} \left( \frac{[t] + u}{T} \right) \cos \left[ \omega_0 ([t] + u) + \epsilon ([t] + u)^2 \right] \right. \\ & \left. - \text{Rect} \left( \frac{[t] - u}{T} \right) \cos \left[ \omega_0 ([t] - u) + \epsilon ([t] - u)^2 \right] \right\}. \end{aligned} \quad (49)$$

Five different states can be recognized from (49) if  $T \geq 2u(u \geq 0)$ .

$$(a) \quad E_a(u, [t]) = 0, \quad \text{for} \quad [t] \leq -(T/2 + u). \quad (50a)$$

$$(b) \quad E_b(u, [t]) = \frac{\mu_0 a}{8\pi R u} \cos \left[ \omega_0 ([t] + u) + \epsilon ([t] + u)^2 \right], \quad (50b)$$

for  $-(T/2 + u) \leq [t] \leq -(T/2 - u)$ .

$$(c) \quad E_c(u, [t]) = -\frac{\mu_0 a}{4\pi R u} \sin(\omega_0 + 2\epsilon[t])u \cdot \sin(\omega_0[t] + \epsilon[t]^2 + \epsilon u^2), \quad (50c)$$

for  $-(T/2 - u) \leq [t] \leq (T/2 - u)$ .

$$(d) \quad E_d(u, [t]) = -\frac{\mu_0 a}{8\pi R u} \cos \left[ \omega_0 ([t] - u) + \epsilon ([t] - u)^2 \right], \quad (50d)$$

for  $(T/2 - u) \leq [t] \leq (T/2 + u)$ .

$$(e) \quad E_e(u, [t]) = 0, \quad \text{for} \quad [t] \geq (T/2 + u). \quad (50e)$$

A graphical interpretation which affords a good overall view of the five states is shown in Fig. 4. It is seen that the time at which a steady

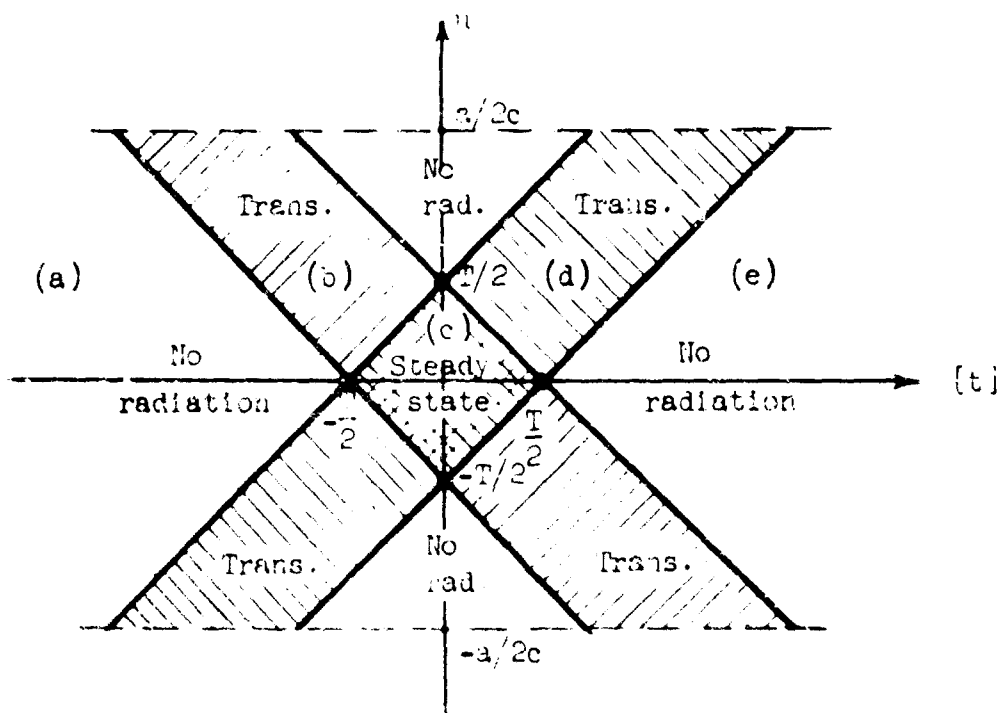


Fig. 4 - Regions of response for uniformly excited aperture to signals in a pulse-compression system.

state is established and the duration of the steady state depend on the value of  $u$  (and therefore  $\theta$ ). When  $u > T/2$ , or  $T < 2u$ , the steady state is never reached and the response simply consists of two regions of transient states. The figure also enables us to visualize the state of response of the antenna in various directions for a given value of  $[t] = t - R/c$ . The lower half of Fig. 4 is for negative values of  $u$  and is obtained easily by an extension of the upper half in accordance with (46).

### VIII. CONCLUSIONS

A simplified method for the analytical determination of the transient and steady-state pattern characteristics of an antenna in response to arbitrary time signals has been presented. In the case of a uniformly illuminated aperture, the present formulation is particularly advantageous, being able to yield the desired response through simple shifts of the signal regardless of how complicated the latter is. No integrations are involved and the overall behavior is easily interpreted. The method is, however, not limited to the case of uniformly illuminated apertures. Convenient graphical representations indicating the dependence of the transient and steady-state regions upon time, distance, direction, aperture dimension, and pulse duration are also given.

In general, for suddenly applied excitations, there is a region of no radiation until  $t = (R - \frac{a}{2} \sin \theta)/c$  when the transient state begins. The transient state goes on for a period equal to  $2u = (a/c)\sin \theta$  and the steady state is established at the end of this period. For time signals with a rectangular envelope (rectangular pulses), there are five different states for the response if the duration of the pulse,  $T$ , is greater than

or equal to  $2u$ . The steady state lasts for a period equal to  $(T - 2u)$ . If  $T < 2u$ , a steady state is never established. When the aperture dimension,  $a$ , is known, the width of the main beam ( $2\theta_1$ , if  $\theta_1$  denotes the location of the first nulls) can be calculated, which in turn specifies the minimum pulse duration,  $2u_1$ , (and the best attainable range resolution) for the establishment of a steady-state main beam.

The response of a discrete, phased array to a signal for electronic scanning has been studied and it was possible to determine the maximum rate of scan in order to avoid pattern distortion. A detailed analysis of the different regions of pattern response to a time signal in a typical pulse-compression system has also been given, which would be extremely difficult to handle without the method presented herein.



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